

Variation partitioning for four explanatory data tables --
 Table 1 with m1 variables, Table 2 with m2 variables, Table3 with m3 variables, Table4 with m4 variables
 Number of fractions: 16, called [a] ... [p].
 √ indicates the 15 regression or canonical analyses that have to be computed.

Compute	Fitted	Residuals	Derived fractions	Degrees of freedom
Direct canonical analysis				
√ Y.1	[a+e+g+h+k+l+n+o]	[b+c+d+f+i+j+m+p]	(1)	df(a+e+g+h+k+l+n+o) = m1
√ Y.2	[b+e+f+i+k+l+m+o]	[a+c+d+g+h+j+n+p]	(2)	df(b+e+f+i+k+l+m+o) = m2
√ Y.3	[c+f+g+j+l+m+n+o]	[a+b+d+e+h+i+k+p]	(3)	df(c+f+g+j+l+m+n+o) = m3
√ Y.4	[d+h+i+j+k+m+n+o]	[a+b+c+e+f+g+l+p]	(4)	df(d+h+i+j+k+m+n+o) = m4
√ Y.1,2	[a+b+e+f+g+h+i+k+l+m+n+o]	[c+d+j+p]	(5)	df(a+b+e+f+g+h+i+k+l+m+n+o) = m5 ≤ m1+m2
√ Y.1,3	[a+c+e+f+g+h+j+k+l+m+n+o]	[b+d+i+p]	(6)	df(a+c+e+f+g+h+j+k+l+m+n+o) = m6 ≤ m1+m3
√ Y.1,4	[a+d+e+g+h+i+j+k+l+m+n+o]	[b+c+f+p]	(7)	df(a+d+e+g+h+i+j+k+l+m+n+o) = m7 ≤ m1+m4
√ Y.2,3	[b+c+e+f+g+i+j+k+l+m+n+o]	[a+d+h+p]	(8)	df(b+c+e+f+g+i+j+k+l+m+n+o) = m8 ≤ m2+m3
√ Y.2,4	[b+d+e+f+h+i+j+k+l+m+n+o]	[a+c+g+p]	(9)	df(b+d+e+f+h+i+j+k+l+m+n+o) = m9 ≤ m2+m4
√ Y.3,4	[c+d+f+g+h+i+j+k+l+m+n+o]	[a+b+e+p]	(10)	df(c+d+f+g+h+i+j+k+l+m+n+o) = m10 ≤ m3+m4
√ Y.1,2,3	[a+b+c+e+f+g+h+i+j+k+l+m+n+o]	[d+p]	(11)	df(a+b+c+e+f+g+h+i+j+k+l+m+n+o) = m11 ≤ m1+m2+m3
√ Y.1,2,4	[a+b+d+e+f+g+h+i+j+k+l+m+n+o]	[c+p]	(12)	df(a+b+d+e+f+g+h+i+j+k+l+m+n+o) = m12 ≤ m1+m2+m4
√ Y.1,3,4	[a+c+d+e+f+g+h+i+j+k+l+m+n+o]	[b+p]	(13)	df(a+c+d+e+f+g+h+i+j+k+l+m+n+o) = m13 ≤ m1+m3+m4
√ Y.2,3,4	[b+c+d+e+f+g+h+i+j+k+l+m+n+o]	[a+p]	(14)	df(b+c+d+e+f+g+h+i+j+k+l+m+n+o) = m14 ≤ m2+m3+m4
√ Y.1,2,3,4	[a+b+c+d+e+f+g+h+i+j+k+l+m+n+o]	[p]	(15)	df(a+b+c+d+e+f+g+h+i+j+k+l+m+n+o) = m15 ≤ m1+m2+m3+m4
Partial analyses				
controlling for one table X				
		(16) [a+g+h+n]	= [a+b+e+f+g+h+i+k+l+m+n+o] - [b+e+f+i+k+l+m+o]	df(a+g+h+n) = m5 - m2
		(17) [a+e+h+k]	= [a+c+e+f+g+h+j+k+l+m+n+o] - [c+f+g+j+l+m+n+o]	df(a+e+h+k) = m6 - m3
		(18) [a+e+g+l]	= [a+d+e+g+h+i+j+k+l+m+n+o] - [d+h+i+j+k+m+n+o]	df(a+e+g+l) = m7 - m4
		(19) [b+f+i+m]	= [a+b+e+f+g+h+i+k+l+m+n+o] - [a+e+g+h+k+l+n+o]	df(b+f+i+m) = m5 - m1
		(20) [b+e+i+k]	= [b+c+e+f+g+i+j+k+l+m+n+o] - [c+f+g+j+l+m+n+o]	df(b+e+i+k) = m8 - m3
		(21) [b+e+f+l]	= [b+d+e+f+h+i+j+k+l+m+n+o] - [d+h+i+j+k+m+n+o]	df(b+e+f+l) = m9 - m4
		(22) [c+f+j+m]	= [a+c+e+f+g+h+j+k+l+m+n+o] - [a+e+g+h+k+l+n+o]	df(a) = m6 - m1
		(23) [c+g+j+n]	= [b+c+e+f+g+i+j+k+l+m+n+o] - [b+e+f+i+k+l+m+o]	df(a) = m8 - m2
		(24) [c+f+g+l]	= [c+d+f+g+h+i+j+k+l+m+n+o] - [d+h+i+j+k+m+n+o]	df(a) = m10 - m4
		(25) [d+i+j+m]	= [a+d+e+g+h+i+j+k+l+m+n+o] - [a+e+g+h+k+l+n+o]	df(a) = m7 - m1
		(26) [d+h+j+n]	= [b+d+e+f+h+i+j+k+l+m+n+o] - [b+e+f+i+k+l+m+o]	df(a) = m9 - m2
		(27) [d+h+i+k]	= [c+d+f+g+h+i+j+k+l+m+n+o] - [c+f+g+j+l+m+n+o]	df(a) = m10 - m3
controlling for two tables X				
		(28) [a+e]	= [a+c+d+e+f+g+h+i+j+k+l+m+n+o] - [c+d+f+g+h+i+j+k+l+m+n+o]	df(a+e) = m13 - m10
		(29) [a+g]	= [a+b+d+e+f+g+h+i+j+k+l+m+n+o] - [b+d+e+f+h+i+j+k+l+m+n+o]	df(a+g) = m12 - m9
		(30) [a+h]	= [a+b+c+e+f+g+h+i+j+k+l+m+n+o] - [b+c+e+f+g+i+j+k+l+m+n+o]	df(a+h) = m11 - m8
		(31) [b+e]	= [b+c+d+e+f+g+h+i+j+k+l+m+n+o] - [c+d+f+g+h+i+j+k+l+m+n+o]	df(b+e) = m14 - m10
		(32) [b+f]	= [a+b+d+e+f+g+h+i+j+k+l+m+n+o] - [a+d+e+g+h+i+j+k+l+m+n+o]	df(b+f) = m12 - m7
		(33) [b+i]	= [a+b+c+e+f+g+h+i+j+k+l+m+n+o] - [a+c+e+f+g+h+j+k+l+m+n+o]	df(b+i) = m11 - m6
		(34) [c+f]	= [a+c+d+e+f+g+h+i+j+k+l+m+n+o] - [a+d+e+g+h+i+j+k+l+m+n+o]	df(c+f) = m13 - m7
		(35) [c+g]	= [b+c+d+e+f+g+h+i+j+k+l+m+n+o] - [b+d+e+f+h+i+j+k+l+m+n+o]	df(c+g) = m14 - m9
		(36) [c+j]	= [a+b+c+e+f+g+h+i+j+k+l+m+n+o] - [a+b+e+f+g+h+i+k+l+m+n+o]	df(c+j) = m11 - m5
		(37) [d+h]	= [b+c+d+e+f+g+h+i+j+k+l+m+n+o] - [b+c+e+f+g+i+j+k+l+m+n+o]	df(d+h) = m14 - m8
		(38) [d+i]	= [a+c+d+e+f+g+h+i+j+k+l+m+n+o] - [a+c+e+f+g+h+j+k+l+m+n+o]	df(d+i) = m13 - m6
		(39) [d+j]	= [a+b+d+e+f+g+h+i+j+k+l+m+n+o] - [a+b+e+f+g+h+i+k+l+m+n+o]	df(d+j) = m12 - m5
controlling for three tables X				
		(40) [a]	= [a+b+c+d+e+f+g+h+i+j+k+l+m+n+o] - [b+c+d+e+f+g+h+i+j+k+l+m+n+o]	df(a) = m15 - m14
		(41) [b]	= [a+b+c+d+e+f+g+h+i+j+k+l+m+n+o] - [a+c+d+e+f+g+h+i+j+k+l+m+n+o]	df(b) = m15 - m13
		(42) [c]	= [a+b+c+d+e+f+g+h+i+j+k+l+m+n+o] - [a+b+d+e+f+g+h+i+j+k+l+m+n+o]	df(c) = m15 - m12
		(43) [d]	= [a+b+c+d+e+f+g+h+i+j+k+l+m+n+o] - [a+b+c+e+f+g+h+i+j+k+l+m+n+o]	df(d) = m15 - m11
Fractions estimated by subtraction (cannot be tested)				
		(44) [e]	= [a+e] - [a]	df(e) = m1-m1 = 0
		(45) [f]	= [b+f] - [b]	df(f) = m2-m2 = 0
		(46) [g]	= [a+g] - [a]	df(g) = m1-m1 = 0
		(47) [h]	= [a+h] - [a]	df(h) = m1-m1 = 0
		(48) [i]	= [b+i] - [b]	df(i) = m2-m2 = 0
		(49) [j]	= [c+j] - [c]	df(j) = m3-m3 = 0
		(50) [k]	= [a+e+h+k] - [a+e] - [h]	df(k) = m1-m1-0 = 0
		(51) [l]	= [a+e+g+l] - [a+e] - [g]	df(l) = m1-m1-0 = 0
		(52) [m]	= [b+f+i+m] - [b+f] - [i]	df(m) = m2-m2-0 = 0
		(53) [n]	= [a+g+h+n] - [a+g] - [h]	df(n) = m1-m1-0 = 0
		(54) [o]	= [a+e+g+h+k+l+n+o] - [a+e+h+k] - [g] - [l] - [n]	df(o) = m1-m1-0-0-0 = 0
		(55) [p]	= residuals = 1 - [a+b+c+d+e+f+g+h+i+j+k+l+m+n+o]	df2(p) = n-1-m15

Tests of significance --

$F(a+e+g+h+k+l+n+o) = ([a+e+g+h+k+l+n+o]/m1)/([b+c+d+f+i+j+m+p]/(n-1-m1))$
 $F(b+e+f+i+k+l+m+o) = ([b+e+f+i+k+l+m+o]/m2)/([a+c+d+g+h+j+n+p]/(n-1-m2))$
 $F(c+f+g+j+l+m+n+o) = ([c+f+g+j+l+m+n+o]/m3)/([a+b+d+e+h+i+k+p]/(n-1-m3))$
 $F(d+h+i+j+k+m+n+o) = ([d+h+i+j+k+m+n+o]/m4)/([a+b+c+e+f+g+l+p]/(n-1-m4))$
 $F(a+b+e+f+g+h+i+k+l+m+n+o) = ([a+b+e+f+g+h+i+k+l+m+n+o]/m5)/([c+d+j+p]/(n-1-m5))$
 $F(a+c+e+f+g+h+j+k+l+m+n+o) = ([a+c+e+f+g+h+j+k+l+m+n+o]/m6)/([b+d+i+p]/(n-1-m6))$
 $F(a+d+e+g+h+i+j+k+l+m+n+o) = ([a+d+e+g+h+i+j+k+l+m+n+o]/m7)/([b+c+f+p]/(n-1-m7))$
 $F(b+c+e+f+g+i+j+k+l+m+n+o) = ([b+c+e+f+g+i+j+k+l+m+n+o]/m8)/([a+d+h+p]/(n-1-m8))$
 $F(b+d+e+f+h+i+j+k+l+m+n+o) = ([b+d+e+f+h+i+j+k+l+m+n+o]/m9)/([a+c+g+p]/(n-1-m9))$
 $F(c+d+f+g+h+i+j+k+l+m+n+o) = ([c+d+f+g+h+i+j+k+l+m+n+o]/m10)/([a+b+e+p]/(n-1-m10))$
 $F(a+b+c+e+f+g+h+i+j+k+l+m+n+o) = ([a+b+c+e+f+g+h+i+j+k+l+m+n+o]/m11)/([d+p]/(n-1-m11))$
 $F(a+b+d+e+f+g+h+i+j+k+l+m+n+o) = ([a+b+d+e+f+g+h+i+j+k+l+m+n+o]/m12)/([c+p]/(n-1-m12))$
 $F(a+c+d+e+f+g+h+i+j+k+l+m+n+o) = ([a+c+d+e+f+g+h+i+j+k+l+m+n+o]/m13)/([b+p]/(n-1-m13))$
 $F(b+c+d+e+f+g+h+i+j+k+l+m+n+o) = ([b+c+d+e+f+g+h+i+j+k+l+m+n+o]/m14)/([a+p]/(n-1-m14))$
 $F(a+b+c+d+e+f+g+h+i+j+k+l+m+n+o) = ([a+b+c+d+e+f+g+h+i+j+k+l+m+n+o]/m15)/([p]/(n-1-m15))$

$F(a+g+h+n) = ([a+g+h+n]/(m5-m2))/([c+d+j+p]/(n-1-m5))$
For the other fractions controlling for one table X, the F-statistics are constructed in the same way

$F(a+e) = ([a+e]/(m13-m10))/([b+p]/(n-1-m13))$
For the other fractions controlling for two tables X, the F-statistics are constructed in the same way

Fractions controlling for three tables X:
 $F(a) = ([a]/(m15-m14))/([p]/(n-1-m15))$
 $F(b) = ([b]/(m15-m13))/([p]/(n-1-m15))$
 $F(c) = ([c]/(m15-m12))/([p]/(n-1-m15))$
 $F(d) = ([d]/(m15-m11))/([p]/(n-1-m15))$

Other fractions combining elementary fractions [a] to [o] can be calculated, but cannot be tested because they cannot be obtained by regression.
