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Variation partitioning for two explanatory data tables --
Table 1 with m1 explanatory variables, Table 2 with m2 explanatory variables
Number of fractions: 4, called [a] ... [d]
√ indicates the 3 regression or canonical analyses that have to be computed.
# Partial canonical analyses are only computed if tests of significance or biplots are needed.

Compute   Fitted  Residuals   Derived fractions                  Degrees of freedom, numerator of F

√ Y.1     [a+b]     [c+d] (1)                                    df(a+b) = m1
√ Y.2     [b+c]     [a+d] (2)                                    df(b+c) = m2
√ Y.1,2   [a+b+c]    [d]  (3)                                    df(a+b+c) = m3 ≤ m1+m2 (there may be collinearity)
# Y.1|2   [a]        [d]                                         df(a) = m3-m2
# Y.2|1   [c]        [d]                                         df(c) = m3-m1

Partial analyses          (4) [a] = [a+b+c] - [b+c]              df(a) = m3-m2*
controlling for 1 table X (5) [c] = [a+b+c] - [a+b]              df(c) = m3-m1*
                          (6) [b] = [a+b] + [b+c] - [a+b+c]      df(b) = m1+m2-(m1+m2) = 0
                          (7) [d] = residuals = 1 - [a+b+c]      df2(d) = n-1-m3 for denominator of F
* Calculation of d.f. for difference between nested models: see Sokal & Rohlf (1981, 1995) equation 16.14.

Tests of significance --

F(a+b) = ([a+b]/m1)/([c+d]/(n-1-m1))
F(b+c) = ([b+c]/m2)/([a+d]/(n-1-m2))
F(a+b+c) = ([a+b+c]/m3)/([d]/(n-1-m3))

F(a) = ([a]/(m3-m2))/([d]/(n-1-m3))
F(c) = ([c]/(m3-m1))/([d]/(n-1-m3))

The only testable fractions are those that can be obtained directly by regression or canonical analysis.
The non-testable fraction is [b]. That fraction cannot be obtained directly by regression or canonical analysis.

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Variation partitioning for three explanatory data tables --
Table 1 with m1 explanatory variables, Table 2 with m2 explanatory variables, Table3 with m3 explanatory variables
Number of fractions: 8, called [a] ... [h]
√ indicates the 7 regression or canonical analyses that have to be computed.
# Partial canonical analyses are only computed if tests of significance or biplots are needed.

Compute   Fitted       Residuals  Derived fractions                        Degrees of freedom, numerator of F

Direct canonical analysis
√ Y.1     [a+d+f+g]     [b+c+e+h] (1)                                      df(a+d+f+g) = m1
√ Y.2     [b+d+e+g]     [a+c+f+h] (2)                                      df(b+d+e+g) = m2
√ Y.3     [c+e+f+g]     [a+b+d+h] (3)                                      df(c+e+f+g) = m3
√ Y.1,2   [a+b+d+e+f+g]     [c+h] (4)                                      df(a+b+d+e+f+g) = m4 ≤ m1+m2 (collinearity?)
√ Y.1,3   [a+c+d+e+f+g]     [b+h] (5)                                      df(a+c+d+e+f+g) = m5 ≤ m1+m3 (collinearity?)
√ Y.2,3   [b+c+d+e+f+g]     [a+h] (6)                                      df(b+c+d+e+f+g) = m6 ≤ m2+m3 (collinearity?)
√ Y.1,2,3 [a+b+c+d+e+f+g]     [h] (7)                                      df(a+b+c+d+e+f+g) = m7 ≤ m1+m2+m3 (collinearity?)
# Y.1|2   [a+f]             [c+h]                                          df(a+f) = m4-m2
# Y.1|3   [a+d]             [b+h]                                          df(a+d) = m5-m3
# Y.2|1   [b+e]             [c+h]                                          df(b+e) = m4-m1
# Y.2|3   [b+d]             [a+h]                                          df(b+d) = m6-m3
# Y.3|1   [c+e]             [b+h]                                          df(c+e) = m5-m1
# Y.3|2   [c+f]             [a+h]                                          df(c+f) = m6-m2
# Y.1|2,3 [a]                 [h]                                          df(a) = m7-m6
# Y.2|1,3 [b]                 [h]                                          df(b) = m7-m5
# Y.3|1,2 [c]                 [h]                                          df(c) = m7-m4

Partial analyses               (8) [a] = [a+b+c+d+e+f+g] - [b+c+d+e+f+g]    df(a) = m7-m6
controlling for two tables X   (9) [b] = [a+b+c+d+e+f+g] - [a+c+d+e+f+g]    df(b) = m7-m5
                              (10) [c] = [a+b+c+d+e+f+g] - [a+b+d+e+f+g]    df(c) = m7-m4

controlling for one table X   (11) [a+d] = [a+c+d+e+f+g] - [c+e+f+g]        df(a+d) = m5-m3
                              (12) [a+f] = [a+b+d+e+f+g] - [b+d+e+g]        df(a+f) = m4-m2
                              (13) [b+d] = [b+c+d+e+f+g] - [c+e+f+g]        df(b+d) = m6-m3
                              (14) [b+e] = [a+b+d+e+f+g] - [a+d+f+g]        df(b+e) = m4-m1
                              (15) [c+e] = [a+c+d+e+f+g] - [a+d+f+g]        df(c+e) = m5-m1
                              (16) [c+f] = [b+c+d+e+f+g] - [b+d+e+g]        df(c+f) = m6-m2

Fractions estimated           (17) [d] = [a+d] - [a]                        df(d) = m1-m1 = 0
by subtraction                (18) [e] = [b+e] - [b]                        df(e) = m2-m2 = 0
(cannot be tested)            (19) [f] = [c+f] - [c]                        df(f) = m3-m3 = 0
                              (20) [g] = [a+b+c+d+e+f+g]-[a+d]-[b+e]-[c+f]  df(g) = (m1+m2+m3)-m1-m2-m3 = 0
                                or [g] = [a+d+f+g] - [a] - [d] - [f]        df(g) = m1-m1-0-0 = 0
                              (21) [h] = residuals = 1 - [a+b+c+d+e+f+g]    df2(h) = n-1-m7 for denominator of F

Tests of significance --

F(a+d+f+g) = ([a+d+f+g]/m1)/([b+c+e+h]/(n-1-m1))
F(b+d+e+g) = ([b+d+e+g]/m2)/([a+c+f+h]/(n-1-m2))
F(c+e+f+g) = ([c+e+f+g]/m3)/([a+b+d+h]/(n-1-m3))
F(a+b+d+e+f+g) = ([a+b+d+e+f+g]/m4)/([c+h]/(n-1-m4))
F(a+c+d+e+f+g) = ([a+c+d+e+f+g]/m5)/([b+h]/(n-1-m5))
F(b+c+d+e+f+g) = ([b+c+d+e+f+g]/m6)/([a+h]/(n-1-m6))
F(a+b+c+d+e+f+g) = ([a+b+c+d+e+f+g]/m7)/([h]/(n-1-m7))

F(a) = ([a]/(m7-m6))/([h]/(n-1-m7))
F(b) = ([b]/(m7-m5))/([h]/(n-1-m7))
F(c) = ([c]/(m7-m4))/([h]/(n-1-m7))
F(a+d) = ([a+d]/(m5-m3))/([b+h]/(n-1-m5))
F(a+f) = ([a+f]/(m4-m2))/([c+h]/(n-1-m4))
F(b+d) = ([b+d]/(m6-m3))/([a+h]/(n-1-m6))
F(b+e) = ([b+e]/(m4-m1))/([c+h]/(n-1-m4))
F(c+e) = ([c+e]/(m5-m1))/([b+h]/(n-1-m5))
F(c+f) = ([c+f]/(m6-m2))/([a+h]/(n-1-m6))

The only testable fractions are those that can be obtained directly by regression or canonical analysis.
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